FLOW

flow through network?

def Directed graph $G=(V,E)$ is simple, $\begin{cases} \text{no self loop} \\ \text{no isolated vertex} \\ \text{no parallel edges.} \end{cases}$

Will allow antiparallel.

def Capacitated graph.

simple directed graph

w/ edge capacity $(C_e)_{e\in E}$, $C_e \in \mathbb{N}$

Notation

for $s,t \in V$, (s,t) flow in G is $f=(f_e)_{e\in E}$, $f_e \in \mathbb{R}_{\geq 0}$

$0 \leq f_e \leq C_e$, $\forall e$.

Def Conservation

$$f^{in}(v) = \sum_{e: u \to v} f_e \qquad f^{out}(v) = \sum_{e: v \to u} f_e$$

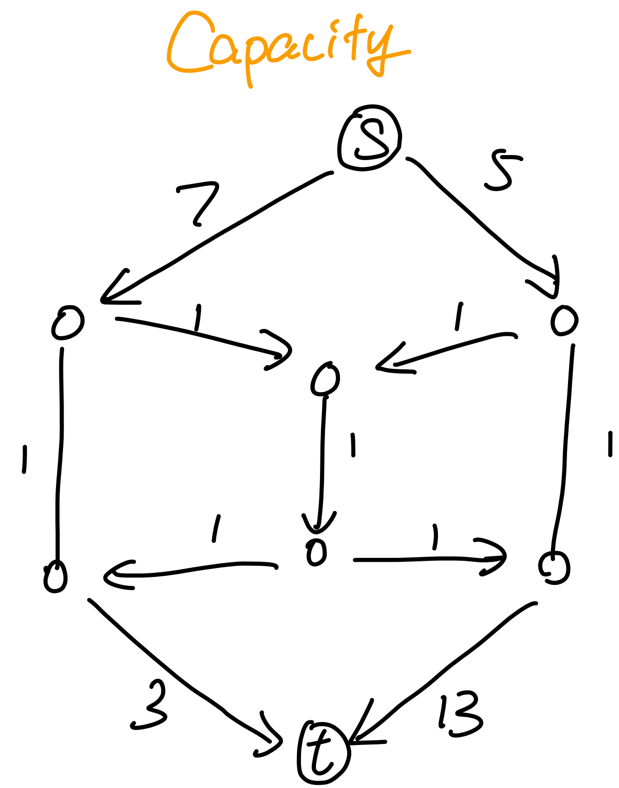
$$f(v) = f^{out}(v) - f^{in}(v)$$

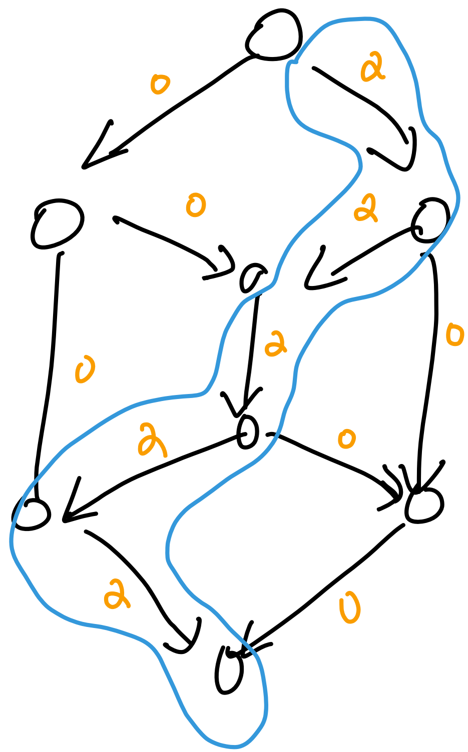
net flow: $\forall v \in V \setminus \{s,t\}$, $f(v) = 0$

value of flow: $|f| = f(s)$

Maxflow is compute $\arg \max\limits_{f(s,t)\text{ flow}} |f|$

Example





If $\alpha = \pi - 3$,

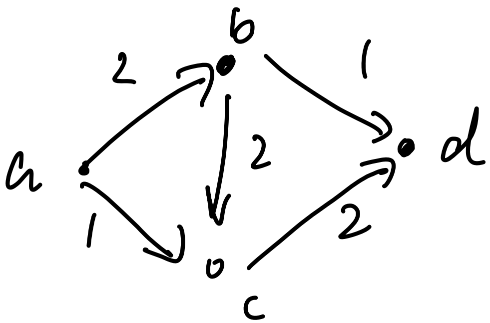
value of flow is $\alpha = \pi - 3$

Cor max flow can be computed

Pf sketch: only finite # of integer flow.

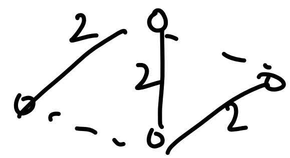
Q efficiently? { DP? / greedy?

Greedy?

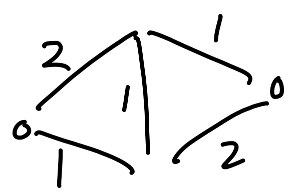


idea: push along s→t path

Local max: one path a→b→d has capacity 2. So $|f|=2$



Global max:



$|f|=3$.

we can also push backwards.

def

$G^f = (V^f, E^f)$ residue graph

- $V^f = V$

- $E^f = \{e : e \in E,\ f_e < c_e\}$ ← forward edge

$\quad \cup \{\underline{-e} : e \in E,\ 0 < f_e\}$ ← backward edge

↑ reversed

Then residue capacity, $e \in E^f$, $(c^f)_e = \begin{cases} c_e - f_e > 0 & e \text{ forward} \\ f_{-e} > 0 & e \text{ backward.} \end{cases}$

def