

Max flow

def Capacitated graph

Conservation

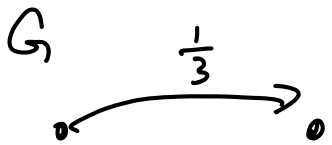
Residue graph

augmented path

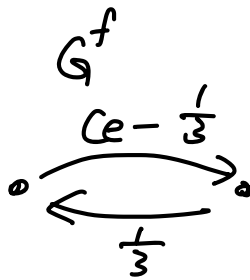
idea:

- $f_e = 0 \quad \forall e \in E$

- repeatedly push new flows along $s \rightarrow t$ paths. Allow negative flows.



residue graph



algo (Ford Folk..)

- $f_e \leq 0, \forall e \in E$
- init G^f
- while exists $p: s \rightarrow t$ in G^f ,
 - augment G
 - update G^f

Correctness none trivial.

def G capacitated, $s, t \in V$, (s, t) cut C
is a partition $V = S \cup T$, $s \in S$, $t \in T$.

Capacity of the cut $|C| = \sum_{e: u \rightarrow v} c_e$, $u \in S$, $v \in T$.

Min Cut!

def f is a flow in G , $S \subseteq V$, flow through S .

$$f^{\text{out}}(S) = \sum_{e: u \rightarrow v} f_e, \quad u \in S, v \notin S$$

$$f^{\text{in}}(S) = \sum_{e: u \leftarrow v} f_e, \quad u \in S, v \notin S$$

Thm

$$\max_{f(s,t): \text{flow}} |f| \leq \min_{(S,t) \text{ cut}} |C|$$

Cor

Given f, C , $|f| = |C|$, then
 f is max flow, C is min cut.