prop

f flow in G, $S = \{v : s \to v \text{ in } G^f\}$

Then following equiv:

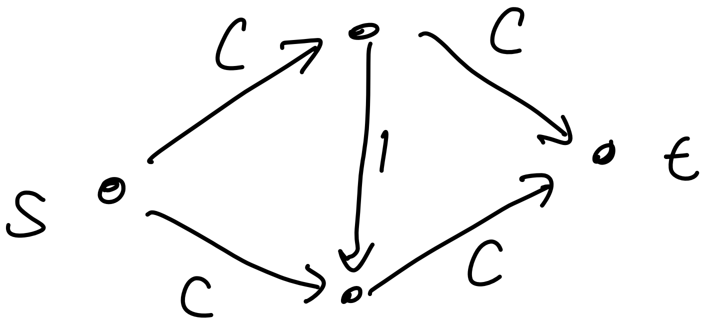
(1) G^f has no $s \rightsquigarrow t$ path

(2) $C = (S, V \setminus S)$ is an (s,t)-cut with $|C| = |f|$

(3) f max flow.

$\exists$ graph, FF runs in $\Omega(F)$

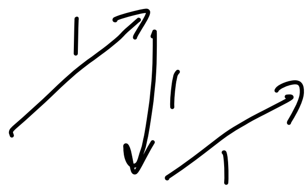
Pf:



$F = \Theta(C)$

flow chart:

①

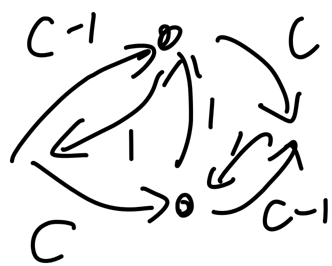


②

⋮

Residual graphs

①



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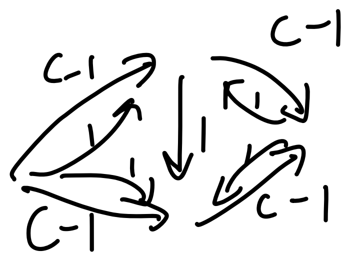


⋮

$\Theta(C)$

By induction.

prop Given flow f in G.

$|f| \geq |f^*| - B$, can find max flow in $O(mB)$

cor: maxflow in $O(m \cdot |f^*|)$ time.

good because $|f^*| \leq \mathrm{poly}(n,m)$.

How to be smart for choosing augmenting path?

idea: want to find large values for path.

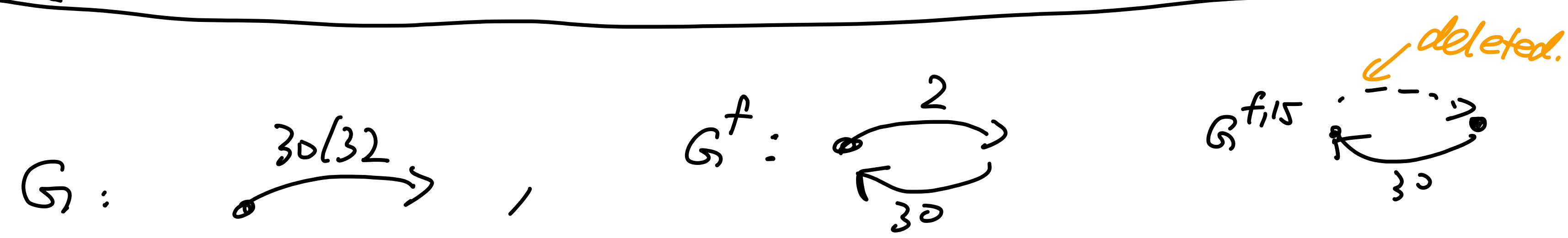
def: f flow in G, $\Delta \in \mathbb{N}_{>0}$, the Δ-bottleneck residual graph $G^{f,\Delta}$

is $V^{f,\Delta} = V$, $E^{f,\Delta} = \{ e \in E^f, (c^f)_e \geq \Delta \}$

$(c^{f,\Delta})_e = (c^f)_e$, $e \in E^{f,\Delta}$

---

e.g.



lemma: $G^{f,1} = G^f$

prop: $s \rightsquigarrow t$ paths in $G^{f,\Delta}$ - are $s \rightsquigarrow t$ path in G^f.

- are augmenting paths w/ value $\geq \Delta$

idea: Start with large Δ, make smaller over time.

Haseva

HASEVA

Haseva

abcdefghi

$\alpha = g^3 0 = 0 - 4 + 5 = 1$

$\alpha = 1$

$g^1 3 = 3 - 22 + 5 = -14 <$

1

$g^{3,1} = 1 - 4 + 5 =$

$f(x,y) =$

$b4 = 4$

if $2 < 4$ h

$\underline{(f\ (a}\ ,\ b)$

fun a $\rightarrow$ fun b $\rightarrow$ (   )

~~a * b~~ $\rightarrow$ (   )

int * int $\rightarrow$ (   )

fun (x, z) $\rightarrow$ (   )

$a = f\ 3$

$5 + 2 + 3 - 5 = 5$

$x = 0$

$f\ 5, 3$